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**Research and Implementation of Path Planning and Trajectory Tracking Algorithm for Monocular Vision Wheeled Robot**

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**ABSTRACT**

Warehouse logistics robots will work in different warehouse environments. In order to enable robots to perceive environment and plan path faster without modifying existing warehouses, we uses monocular camera to achieve an efficient robot system integration. Mapping and path planning the two main tasks presented in this paper. The direct method visual odometry is applied to localize, and the 3D position of major obstacles in the environment is calculated. We describe the terrain with occupied grid map , the 3D points are projected onto the robot motion plane, thus accessibility of each grid is determined. Based on the terrain information, the optimized A\* algorithm is used for path planning. Finally, according to localization and planning, we control the robot track path. We also develop a path-tracking robot prototype. Simulation and experimental results verify the effectiveness and reliability of the proposed method.

**Key Words:** VSLAM, OGM, A\*, Monocular vision, Mobile robot

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# Chapter 1

## 1.1 INTRODUCTION

Windowed Optimization is a classic method in non-linear optimization.

### 1.1.1 NOTATION

Throughout the paper, we will write matrices as bold capital letters () and vectors as bold lower case letters (), light lower-case letters to denote scalars (). Light upper-case letters are used to represent functions ().

Homogeneous camera calibration matrices are denoted by  as (2.1). Camera poses are represented by matrices of the special Euclidean group , which transform a 3D coordinate from the camera coordinate system to the world coordinate system. In this paper, a homogeneous 2D image coordinate point  is represented by its image coordinate and inverse depth as (2.1) relative to its host keyframe . The host keyframe is the frame the point got selected from. Corresponding homogeneous 3D world coordinate point  is denoted as (2.1).  are used to denote camera projection functions. The jacobian of ,  is denoted as (2.1)

### 1.1.2 QUESTION IMPORT

Assume we observe 5 points  in 4 keyframes , every keyframe has stereo vision  abbreviated as . A point can also be observed by other frame as shown in Table(2.1). Question is how to use Windowed Optimization method to make our observation more accurate ?

Table (2.1)

|  |  |  |
| --- | --- | --- |
| Image point | Host keyframe | Observe by |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

## 1.2 SOLUTION

We use direct method to construct residual, Windowed Gauss-Newton method to optimization residual。

### 1.2.1 CONSTRUCT RESIDUAL

Dynamic multi-view stereo residuals  are defined as

 is Huber norm.  is affine brightness parameters to frame  .  is a gradient-dependent weighting parameters,  in frame  projected to  is  as:

Static one-view stereo residuals  are modified to

Hostframe of  is .  is affine brightness parameters to frame .  in frame  projected to  is  as :

Total residuals

To balance the relative weights of temporal multi-view and static stereo, we introduce a coupling factor  to weight the constraints from static stereo differently.  is a set of all image point host by frame .  are the observations of  from temporal multi-view stereo. If there are  image point and  keyframes in , optimization variable  is

In this example, there are 7 dynamic residuals and 3 static residuals, Factor graph of the residuals function is

Total residuals is



Iteration  can be calculated by

We construct residuals and its formulation.

### 1.2.2 JACOBIAN CITATION

 We know for a Lie algebra  and :

### 1.2.3 JACOBIAN DERIVATION

First, if  is neither observed by frame ,  nor hosted by ,  we can get:

otherwise, we follow

For one frame , we have  and , then we can get

add detail scalar derivate…….



## 2.3 图像的深度估计

### 2.3.1 深度均值估计

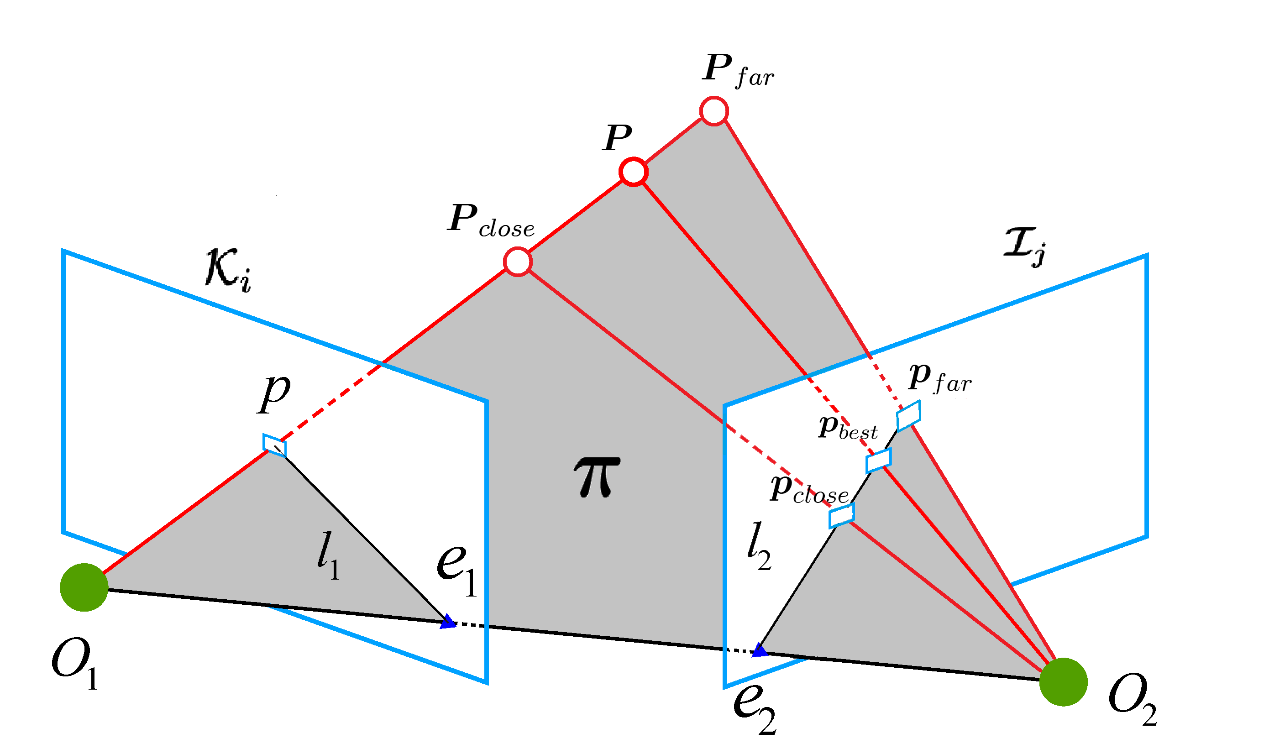


图2.4 对极几何

得到优化的相机位姿以后，可以使用对极几何[71]来初步估计关键帧中集合中点的逆深度。如图2-3所示，分别是帧和帧对应的相机光心位置，为基线(baseline)[60]，与帧和帧的归一化平面(对应相机坐标系的平面)交与极点(epipolar point)和，平面点是空间点(深度未知)投影形成的，三点形成了极平面(epipolar plane)，与帧和帧交于极线(epipolar line)和，上存在一个点由投影形成，根据光度不变假设，，我们在一定范围内去搜索得出，令，当或时，则会形成一个空间点和，经过位姿跟踪得出的和，投影到帧得到和。在极线段寻找一点满足，于是便得到，然后可以算出。详细推导和算法伪代码见附录414。

### 2.3.2 深度方差估计

上述估算深度时，我们使用了两个条件：相机的旋转和平移，块匹配。这两者本身也是估计的，都会带来误差。相机的位姿与相机的朝向和位置有关，而块匹配与图像的灰度值有关。也就是说大致可划分有两种因素对深度估计的均值产生影响，一种是位姿求解和相机标定时带来的几何朝向误差，一种感光器件的噪声引起的光度误差。这两者是相互独立的，两者造成的误差直接作用在极线搜索环节上，极线最佳匹配误差为，对最后深度的影响是间接的，深度的误差，两者成比例关系，的定义如式(2.11)，其中表示每进一个搜索步长对应深度的变化，表示每进一个搜索步长对应极线极线长度的变化。

#### 2.3.2.1 几何误差

在优化位姿得出最优解后，实际的，这就是定位导致的误差。根据附录的式(4.2)中求得了极线的方向向量，当处的梯度向量与平行时，极线搜索的几何误差最小，与的夹角越接近直角，误差越大[15]。几何误差的计算如式(2.12)：

#### 2.3.2.2 光度误差

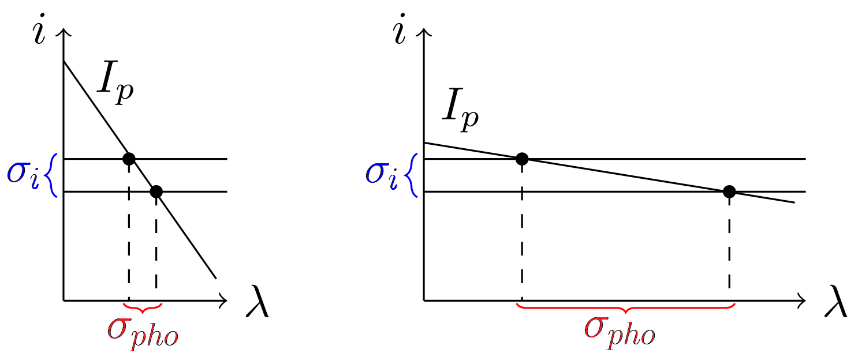
设感光器件的高斯噪声的方差为，为图像沿着极线五个采样点(附录图4-2-b)的灰度梯度，如式(2.13.1)。如图2.5，在同样的器件高斯噪声干扰下，越大，那么在极线上的误差越小。光度误差的计算如式(2.13.1)：

图2.5 对极几何

由上述的几何误差和光度误差可以算出深度的误差，如式(2.13.2)。

### 2.3.3 深度的更新和传播

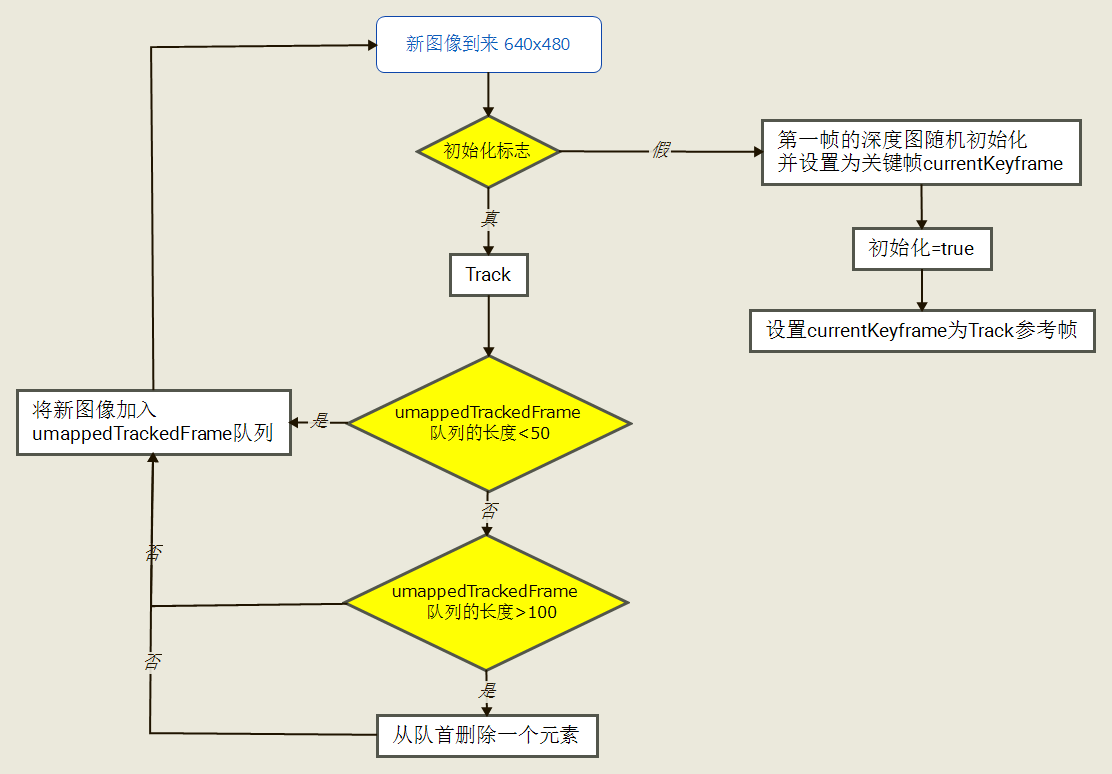
定位和建图分别在两个独立的线程中进行。在定位线程中，第一帧图像到来时，立即将其设为关键帧，并且中各元素的值是随机初始化的，此后多对相机进行平移运动，到来后，首先进行位姿跟踪，得到位姿变换，如此不断跟踪，当相机的运动距离大于阈值时，令createNewKey-Frame标志位为真，建图线程会选定当时这一帧创建新的关键帧。同时在从到这段时间内使用不同的参考帧计算帧的集合中点的深度的均值与方差，建图线程计算速度要比定位线程慢，所以将没有来的及建图的普通帧加入umappedTrackedFrame双端队列中，该队列的长度多于50时，就需要从队首删除元素，定位线程工作流程如图2-6。

图2.6 定位线程

在建图线程中，从umappedTrackedFrame队列中选取以当前关键帧为父帧的普通帧存储referenceFrame队列中，按时间顺序构成集合。先读取关键帧创建标志位，判断是否需要创建关键帧，如果不需要，那么需要为当前关键帧的深度图和在中选择一普通帧作为深度估计的参考帧。

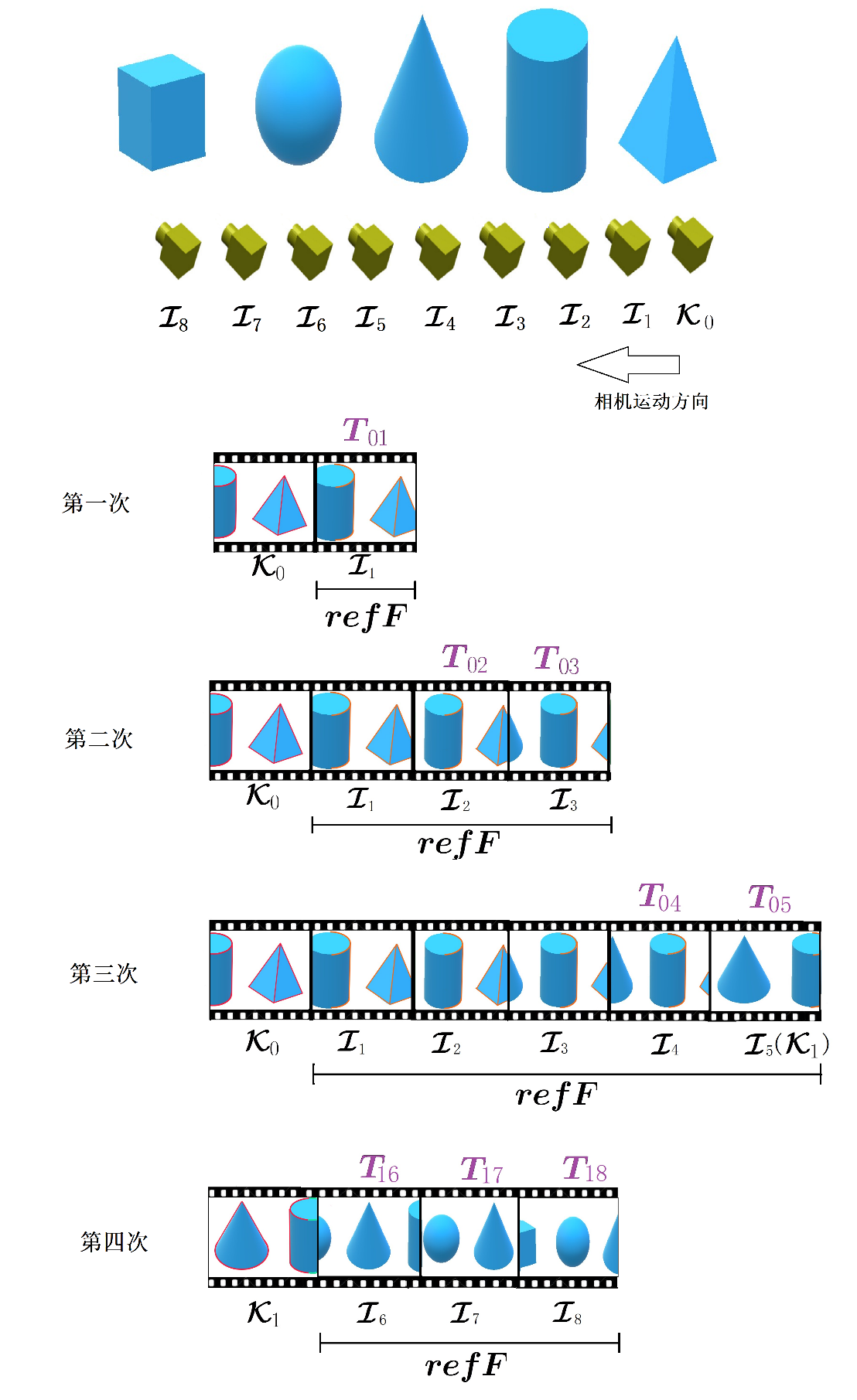
，每个点的参考帧是变化的，根据附录4计算得出极线的长短向后顺延。举个例子，如图2.7，场景中放置着一些几何物体，摄像机从右往左移动，在此过程中拍下了九张照片，，首先被选为关键帧,其中梯度大的像素点加入，，在图中为红色它们的深度均值都为1，方差都为0，此为先验分布，组成了深度图和。

图2.7 建图线程

第一次建图时，referenceFrame中只有，中只有中部分点（橙色色），估计出各个点的后验深度均值与方差，与先验相融合就是两个高斯分

布相乘得到新的分布，如式（2.14）。第二次建图时，referenceFrame中有三帧，在上一次计算中，中的不同点会有不同的极线长度，如果某个点的极线短，就适当往后选择新到的帧作为参考帧，这样又会估计新的后验分布，再次融合。这样就可以在新的关键帧到来之前，不断的调整各个点的深度估计值，会收敛到真实值附近。

第三次建图时，referenceFrame中有五帧，重复之前的深度估计。且此时已经移动了足够的距离，需要新建关键帧。于是选定当前帧作为新的关键帧，中有部分点也在中，用橙色标注。这些点是有先验的深度分布值，根据位姿跟踪的结果将这些值传播至，作为这些点的先验值，其他没有先验的，仍然如先前假设初值，这样就建立起了新的深度图和。

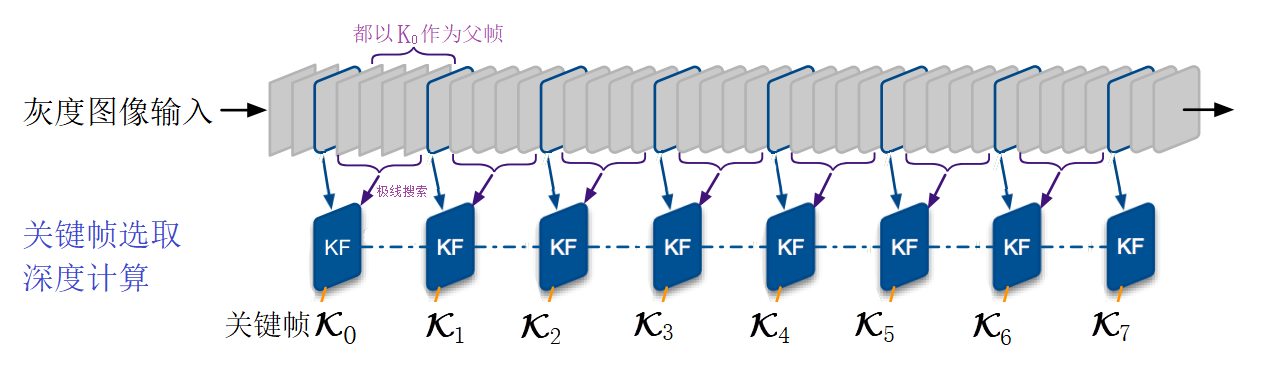
****第四次建图时，中红色是没有被传播深度的点，绿色是被传播深度的点。把referenceFrame中不是以为父帧的删除掉，加入，然后再做深度估计，如此重复，如图2.8所示。

图2.8 不断创建关键帧

传播深度估计时，如式（2.15）分别计算新的深度均值和，其中是相机沿着光轴运动的距离。

## 2.4 本章小结

本节介绍了计算机视觉的基础知识和基于直接法建图和定位的相关理论，包括带权值的位姿优化、梯度点的深度估计与更新传播。为后续章节进行算法框架的搭建、算法改进以及代码编写作理论基础。

# 附录

## 1.带权值的非线性最小二乘方法优化位姿推导和算法：

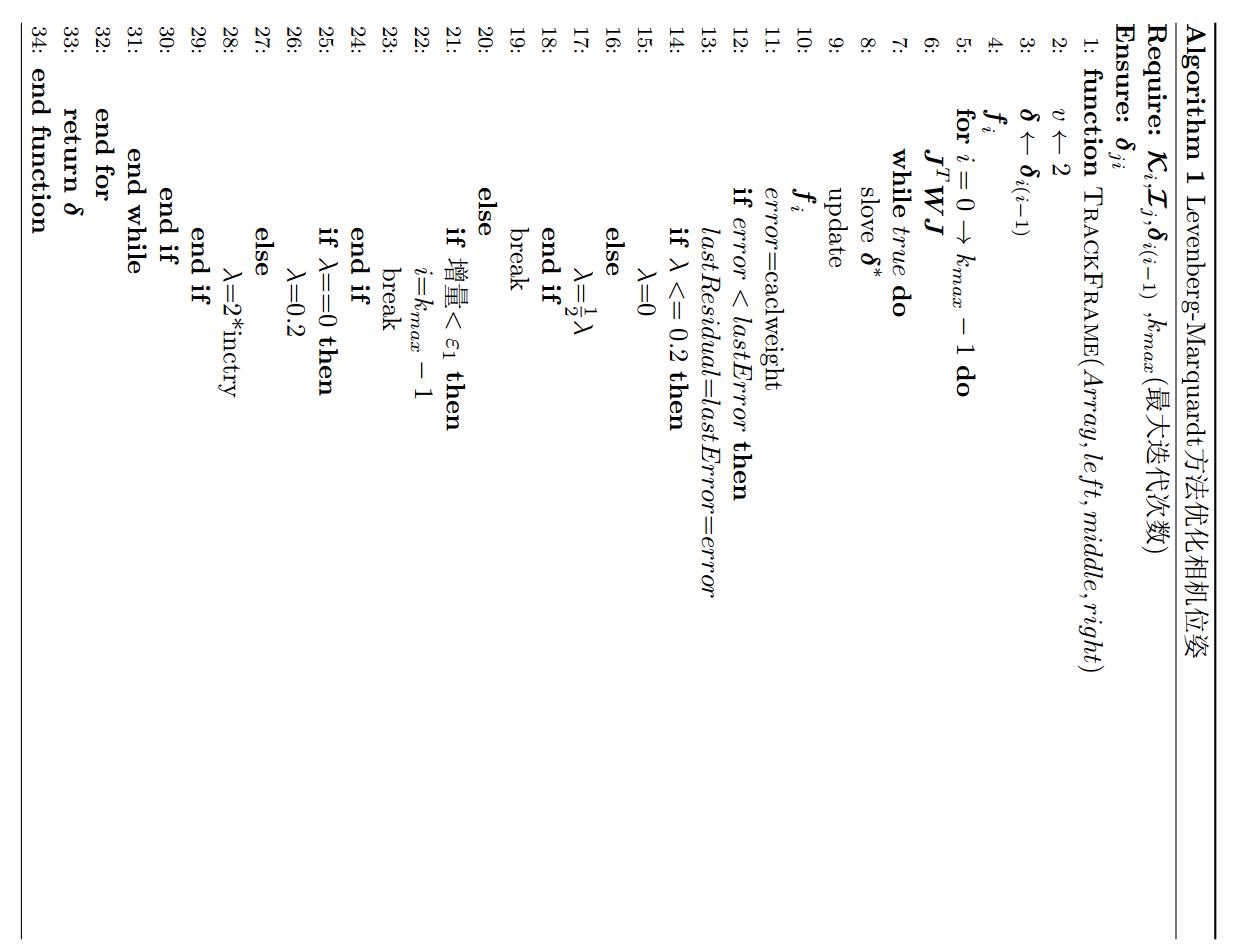
把式2.11.2写成矩阵相乘，再进行一阶泰勒展开，求导，取极值可解得迭代变量，推导如式(1):

如式（2）将个点的雅克比矩阵求到，使用链式求导分三部分求偏导。分别求图像函数对物理平面点的偏导，即梯度；求物理平面点对空间点的偏导；求空间点变换的李代数导数，见式2.8。

再求权值矩阵，如式（3）

最后求解迭代方程，可以证明是对称正定矩阵[69]，遂使用乔列斯基分解[70]可以解得方程，如式（4）：





## 2.极线搜索推导与算法

实际的极线搜索比理论复杂的多。

### 2.1 首先要确定是否能够在极线上搜索到对应的像素点：

图4-1

根据文献[15][72][73]的论证，点匹配结果的好坏与基线的长短有很大关系。图4-1分别来自于上述文章，纵轴是匹配代价函数(cost)，横轴是逆深度值。可以看出当基线长的时候，对应的极线就长，cost随着d增长变化幅度大，有较多的极小值，还需要从这些极小值中分别到底哪个是最佳匹配点。如果选取小基线，虽然函数中仍有很多的极小值，但是存在最小值。

### 2.2其次是使用像素块匹配计算最佳点：

匹配代价函数并不是直接使用两个像素点的灰度值直接相减，而是选取周边的像素值进行插值计算，可以增强鲁棒性。本文使用SSD(差平方和)方法，选取关键帧和待匹配点极线方向上下邻近的五个点，如式(4.1):

### 2.3具体如何操作搜索和计算逆深度：

给定关键帧和参考帧的灰度图，关键帧上待匹配点，关键帧到参考帧位姿变换和。据此可求逆深度:

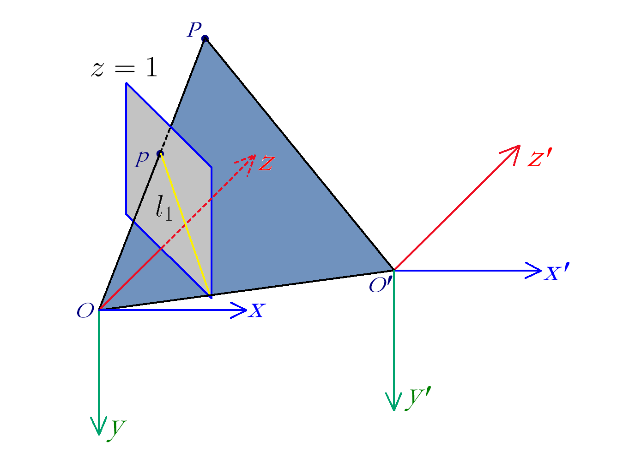
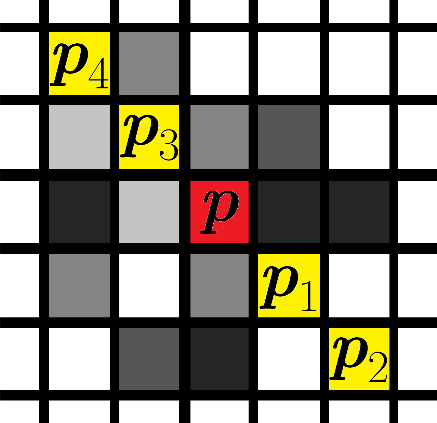
1. 求在归一化平面：上极线方向匹配块：

图4-2 (a) 图4-2 (b)

如图4-2 (a)，连接与像素平面点的射线交的坐标为，和确定极平面，和的法向量分别为，极线的方向向量为：

然后沿着方向取得上下邻近的四个插值亚像素点，组成一个像素块，如图4-2 (b)。

1. 求参考帧上的极线：

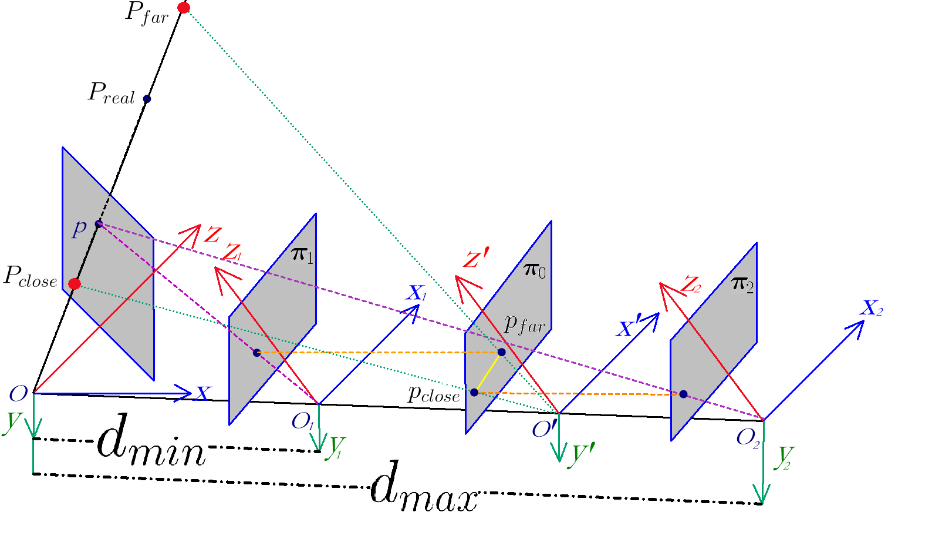
我们并不知道是多少，先假设一个足够大的集合，参考帧上极线形成与相机的平移相关，旋转无关，因为只旋转无法形成基线。如图，可以通过缩放平移矩阵模拟逆深度的变化，从而在构造出极线。如式(4.3)，令，先将旋转，再分别加上逆深度乘以平移。

图4-3

如图4-3，是实际的相机位姿，对应归一化平面，是式4.3.1对应的相机位姿的归一化平面，和互相平行，连接交于，过作的垂线交于点，是形成的极点。同理，可以得到对应的极点，连接就在上形成了一条极线段：。

1. 求极线段上的最佳匹配点

沿着极线段的SSD匹配函数对灰度图做亚像素级别的搜索，搜索间隔是式(4.3.3)中的，为矢量，有纵向和横向的递增距离。将极线使用参数式方程表示，如式(4.4)。找到使得匹配函数的导数为零的极小值集合，求得最优解，该最优解在像素平面上坐标对应平面上的一个点。

1. 求解关键帧上像素点的深度

关键帧上反变换到空间点。假设在参考帧位姿下的深度为。便于书写，令，推导如式(4.5)：

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Chapter1- Chapter3

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